

are first grouped together in a mass, without any subdivision into cortical and medullary portions. On the 14th day the first trace of this subdivision becomes manifest, by the vesicles being aggregated into masses which radiate from the circumference towards the centre of the gland, in some cases complete tubes being formed by the junction of the vesicles, as indicated by hemispherical bulgings along their walls. At a later period the organs increase in size, they attain their usual position, and a more complete subdivision into cortical and medullary portions is now observed.

The author lastly traces out the development of the thyroid glands, and shows the great similarity that exists between them, the spleen and supra-renal glands, from the similar structure they present, and from the development of those structures occurring in a similar manner in each.

These glands are developed between the 6th and 7th days as two separate masses of blastema, one at each side of the root of the neck, close to the separation of the carotid and subclavian vessels, and between the trachea and the branchial clefts, but quite independent, as far as regards their development, of either of those parts. Their minute structure at an early period closely corresponds with that of the spleen and supra-renal glands. Later, when the gland tissue of which the thyroid gland ultimately consists is formed, it is developed in a manner precisely similar to the same tissues of the spleen and supra-renal glands, a fact which shows the analogy they bear to one another.

From these observations the author concludes that a close analogy exists between the glands already described, so that the propriety of their classification under one group, as the "Ductless Glands," may be considered clearly proved. And although the spleen by many has been excluded from them, the author considers that its classification with them is correct, for the following reasons:—1st. From its evolution being similar with that of the supra-renal and thyroid gland; 2nd. from its structure, which at an early period closely corresponds with them; and 3rdly, from the development of its tissues following the same law as that upon which the tissues of the allied glands are formed.

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January 22, 1852.

DR. DAUBENY, V.P., in the Chair.

A paper was read, entitled:—"Researches on the Geometrical Properties of Elliptic Integrals." By the Rev. James Booth, LL.D., F.R.S. &c. Received November 17, 1851.

In this paper the author proposes to investigate the true geometrical basis of that entire class of algebraical expressions, known to mathematicians as elliptic functions or integrals. He sets out by showing what had already been done in this department of the subject by preceding geometers. That the elliptic integral of the

second order represented an arc of a plane ellipse, was evident from the beginning. Hence indeed the name "elliptic functions," derived from a part, was given to the whole. Here then the question naturally arose: What geometrical types did the first and third orders represent? This question long remained without complete solution; and investigators in this department of analysis were compelled to take the fundamental expressions as arbitrary data, and to forgo the inquiry what the geometrical theorems were which these algebraical expressions represented. Various but unsuccessful attempts were made by geometers to represent them by quadratures, or by plane curves, either algebraical or transcendental. About ten years ago, however, Messrs. Guderman and Catalan showed that the circular form of the third order represented the curve of intersection of a cone of the second degree and a concentric sphere; but they did not extend their researches to the first order, nor to the logarithmic form of the third.

The main object of the paper is to prove that elliptic integrals of every kind, the parameter taking any value whatever between positive and negative infinity, represent the intersections of surfaces of the second order.

These surfaces divide themselves into two classes, of which the sphere and the paraboloid are the respective types; from the former arise the circular functions of the third order, from the other the logarithmic and exponential. In the course of these investigations it is shown that the formulæ for the comparison of elliptic integrals, which are given by Legendre, follow simply as geometrical inferences from the fundamental properties of those curves. The ordinary conic sections are merely particular cases of those more general curves, to which the author has given the name Hyperconic Sections.

The author remarks, that it will doubtless appear not a little singular, that the principal properties of these functions, their classification, their transformations, the comparison of elliptic integrals of the third order, with conjugate or reciprocal parameters, were all investigated and developed before geometers had any idea of the true geometrical origin of those functions. It is as if the formulæ of common trigonometry had been derived from an algebraical definition, before the geometrical conception of the circle had been admitted. As trigonometry may be defined, the development of the properties of circular arcs, whether described on a plane, or on the surface of a sphere, so this higher trigonometry, or the theory of elliptic integrals, may be defined as the development of the relations which exist between the arcs of hyperconic sections.

It may be said, we cannot by this method derive any properties of elliptic integrals which may not algebraically be deduced from the fundamental expressions appropriately assumed. It cannot, however, be truly asserted that the properties of curve lines should be developed without any reference to their geometrical types. We might, starting from certain algebraical expressions, derive every known property of curve lines, without having in any instance a conception of the geometrical types which they represent. The

theory of elliptic integrals was developed by a method the inverse of that pursued in establishing the formulæ of common trigonometry. In the latter case, the geometrical type was given—the circle—to determine the algebraical relations of its arcs. In the theory of elliptic integrals, the relations of the arcs of unknown curves are given, to determine the curves themselves; this is the principal object of the present communication.

The problem resolves itself into twelve distinct cases, depending on the magnitude of the parameter, and the sign with which it is affected; out of the discussion of these cases arise many new and important relations of elliptic integrals. It would excite little interest to give the bare enunciations of those theorems, and a mere outline of the methods by which they are established would be unintelligible. Not the least interesting of those theorems is the proposition, that it is always possible to express an elliptic integral of the first order as the sum of two elliptic integrals of the third order, with parameters which are conjugate, reciprocal and imaginary.

The author hopes, in a future communication to the Royal Society,—the present having grown under his hands beyond the limits he anticipated—among other points, to extend his researches to the case of elliptic integrals with imaginary parameters, and to show the true geometrical meaning of such expressions. It will also be shown, that imaginary expressions may be found for a logarithmic elliptic arc analogous to the well-known imaginary exponential expressions for the sines and cosines of circular arcs.

A paper was in part read, entitled, “Further Researches into the Structure, Development and Functions of the Liver.” By C. Handfield Jones, M.D., F.R.S. Received November 19, 1851.

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January 29, 1852.

COLONEL SABINE, R.A., Treasurer, in the Chair.

The reading of Dr. Handfield Jones' paper, “On the Structure of the Liver,” was resumed and concluded.

Dr. Leidy and Professor Retzius, with Muller, Weber and Khronenberg, maintain the existence of plexuses of ducts in the parenchyma of the liver containing the cells in their tubes. Some other anatomists, especially Gerlach, believe the ducts to be prolonged into the lobules of the parenchyma, under the form of mere intercellular passages without walls.

Injections of acetate of lead in saturated solution, thrown into the ductus communis choledochus, produce appearances which seem to confirm the latter view. The author, however, believes them to be fallacious, and that the ducts really terminate, as he has described them in his former paper, by closed extremities, either rounded and even, or